The Richard Stockton College of New Jersey
Mathematical Mayhem 2014
Individual Round - Solutions

March 22, 2014

Name: _____

High School:_____

Instructions:

- This round consists of **18** problems worth a total of **80** points, made up of 8 Appetizers worth 3 points each, 7 Entrées worth 5 points each, and 3 Desserts worth 7 points each.
- Each of the 18 problems is multiple choice, and each problem comes with **5** possible answers.
- For each problem, **circle the best answer**.
- You are not required to show any work this round.
- No calculators are permitted.
- This round is **75 minutes** long. **Good Luck!**

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Problem #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Total
Points Earned																			

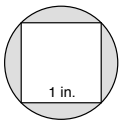
♣ Appetizers ♣

Problem 1. The value of $-1+2-4+8-16-32+64$ is
(A.) -69 (B.) $\boxed{21}$ (C.) 26 (D.) 79 (E.) 127

Problem 2. The expression $(\sqrt{100}+\sqrt{9})\times(\sqrt{100}-\sqrt{9})$ is equal to
(A.) $\boxed{91}$ (B.) 19 (C.) 9991 (D.) 9919 (E.) 10991

Problem 3. If 64 is divided into three parts proportional to 2, 4, and 6, the smallest part is
(A.) $5\frac{1}{3}$ (B.) 11 (C.) $\boxed{10\frac{2}{3}}$ (D.) 5 (E.) none of these answers

Problem 4. In the figure below a square with side length 1 is inscribed in a circle. The area of the shaded region (i.e. the area inside the circle but outside the square) is
(A.) $1/2$ (B.) $\pi/4$ (C.) $1+\pi/2$ (D.) $\boxed{\pi/2-1}$ (E.) $\pi-1$



Problem 5. A collection of coins is made up of an equal number of pennies, nickels, dimes, and quarters. What is the largest possible value of the collection which is less than \$2?
(A.) $\boxed{\$1.64}$ (B.) \$1.78 (C.) \$1.86 (D.) \$1.89 (E.) \$1.99

Problem 6. Reduced to lowest terms, $\frac{a^2-b^2}{ab}-\frac{ab-b^2}{ab-a^2}$ is equal to
(A.) b^2 (B.) $\frac{a^2-2b^2}{ab}$ (C.) a^2 (D.) $a-2b$ (E.) $\boxed{\frac{a}{b}}$

Problem 7. A farmer has geese and donkeys in his barnyard. The farmer counts 24 heads and 68 feet. How many donkeys does the farmer have?
(A.) 8 (B.) 9 (C.) $\boxed{10}$ (D.) 14 (E.) 15

Problem 8. If the digit 1 is placed after a two digit number whose ten's digit is t, and unit's digit is u, the new number is
(A.) $10t+u+1$ (B.) $\boxed{100t+10u+1}$ (C.) $1000t+10u+1$ (D.) $t+u+1$ (E.) $1000t+100u+1$

♢ Entrées ♢

Problem 9. Mathematical Mayhem occurs each year in March. In 40 consecutive months, what is the largest number of Mathematical Mayhem contests that could be held?

(A.) 1 (B.) 2 (C.) 3 (D.) $\boxed{4}$ (E.) 5

Problem 10. If m men can do a job in d days, then $m + r$ men can do the job in

(A.) $d + r$ days (B.) $d - r$ days (C.) $\frac{d}{m+r}$ days (D.) $\boxed{\frac{md}{m+r} \text{ days}}$ (E.) $\frac{m}{m+r}$ days

Problem 11. In the base 10 addition shown below, the letters X, Y, and Z each represent a different non-zero digit (i.e. they are each an integer between 1 and 9) . The digit X is

(A.) 1 (B.) 2 (C.) 7 (D.) $\boxed{8}$ (E.) 9

$$\begin{array}{cccc} & X & X & X \\ & Y & Y & Y \\ + & Z & Z & Z \\ \hline Z & Y & Y & X \end{array}$$

Problem 12. The side, front, and bottom faces of a rectangular box have surface areas 12 square inches, 8 square inches, and 6 square inches respectively. The volume of the box, in cubic inches, is

(A.) 576 (B.) 36 (C.) 9 (D.) 104 (E.) $\boxed{24}$

Problem 13. How many triples (a, b, c) of real numbers satisfy the equations $ab = c$, $ac = b$, and $bc = a$?

(A.)2 (B.) 4 (C.)$\boxed{5}$ (D.) 6 (E.)8

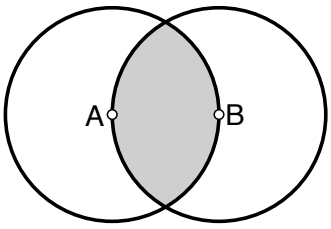
Problem 14. How many of the following numbers are divisible by the sum of their digits?

$$11,\ 111,\ 1111,\ 11111,\ 111111,\ 1111111,\ 11111111,\ 111111111$$

(A.) $\boxed{2}$ (B.) 3 (C.) 4 (D.) 5 (E.) 6

Problem 15. In the figure below, there are two circles of radius 1 with centers A and B. What is the area of the shaded region?

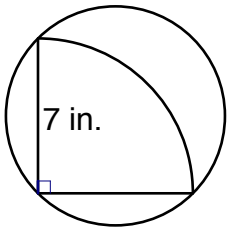
(A.) π (B.) $\pi - \sqrt{2}$ (C.) $\frac{\pi}{2} + \frac{\sqrt{2}}{2}$ (D.) $\pi - \frac{\sqrt{3}}{2}$ (E.) $\boxed{\frac{2\pi}{3} - \frac{\sqrt{3}}{2}}$



♡ Desserts ♡

Problem 16. How many real numbers satisfy the equation $(x^2 + 2x)^{(x^2-3x+2)} = 1$?
(A.) none (B.) 2 (C.)4 (D.) $\boxed{5}$ (E.) infinitely many

Problem 17. Your hungry brother puts $1/4$th of a pizza, with radius 7 inches, on a circular plate. The plate is exactly large enough to hold this oversized "piece" of pizza, as shown in the figure below. What fraction of the plate is covered by the "piece" of pizza?
(A.) 1/4 (B.) 1/3 (C.) $\boxed{1/2}$ (D.) 2/3 (E.) $\sqrt{2}/2$



Problem 18. Consider the pattern ♣♡♢♣♡♡♢♣♡♡♡♢ . . .. Assuming that the pattern continues, how many ♣'s will appear in the first 2014 symbols?
(A.) 61 (B.) 62 (C.) 63 (D.) $\boxed{64}$ (E.) 65